A tool to calculate mechanical forces on railway catenary

J. Benet, F. Cuartero, & T. Rojo
Escuela Politécnica Superior, Univ. de Castilla-La Mancha
Campus, 02071 Albacete, Spain
EMail: {jbenet, fernando, trojo}@info-ab.uclm.es

Abstract

We will present the basis for the mechanical calculation of the catenary system in a railway, considering static wire equations. Our objective is to allow an increment in the speed in railways, by developing an accurate mechanical calculation of the electric wiring known as catenary. Also two algorithms for the calculation are developed. In the first one, the exact weight of the wire is considered, while in the other, the weight of the wire is estimated as a uniform horizontal load. Finally, this algorithm will be used to implement a software tool.

1 Introduction

The evolution in the transport market resulting from the globalization of the economy, the increasing deregulation of the markets, the competition on the base of a customer service, the growing environmental concerns, and the need to ensure a long term operational profitability consistent with prevailing economic reality are driving to a radical change (structural and cultural) in the railway sector, centered on innovative approaches to business and services, conducive to a future-oriented and market-driven posture in the global transportation marketplace.

The survival in the ever evolving and highly competitive transport market, implies a continuous search for the roots of excellence enabling railway operators and supplying industry to achieve a world-class profile. This quest for excellence has to cover the entire range of business activities, beginning with market demand and ending with customer satisfaction. It will entail the need for an integrated development and timely deployment of the adequate organizational, technological and skill infrastructures.

Thus, the fulfillment of this strategy needs new construction concepts, among them, the development of more performant coupled pantograph/ catenary systems and a wide utilization of new technologies.

Furthermore, the multitude of electrification systems currently in use throughout Europe and subsequently the massive capital investment that would be necessary in order to implement any sort of European wide harmonised solution, preclude to envisage any major change in this field in the foreseeable future. A detailed study of electrification systems and catenary/pantograph technology considering economic aspects may be found in [4].

During the recent last years, passengers transportation by railway has experimented a considerable increasing in some European countries (Germany, France, Spain, ...). For that, the reaching of higher velocities in railways has become a very important target. In that scenario, the pantograph/catenary system, with its dynamic behaviour, becomes a crucial component (see [5, 6, 7]), because at high speed it is very difficult to warranty the permanent contact of pantograph head and contact wire, more over without the increasing of noise and wear.

In order to obtain an adequate behaviour in the pantograph/catenary system, it is necessary the existence of adequate conditions in the line, and this requires, among other aspects, a very precise mechanical calculus. Recent investigations have focused on dynamical behaviour by dynamical simulations in order to allow a better interaction of the pantograph and the catenary [7, 3]; in this paper we will follow a more traditional approach, focusing in the catenary, modeled, as usual, by a set of coupled strings.

Of course, the best conditions in which the pantograph would obtain electric energy from the line are when the contact wire is parallel to the ground, and then, an important problem is to determine the exact length of the droppers in order to allow the contact wire to acquire the correct shape. So, our objective is the development of a technique which allows us to implement a high precision calculation algorithm, and thus to develop a software tool to design high quality catenaries.

The paper is structured as follows, section 2 describes a summary of the model of catenaries we are considering. In section 3, it is explained our first algorithm, where we consider the real weight of the carrier, while other approximate version is presented in section 4, considering in this case a distribution of the weight along the length of the gap in place of the real weight obtaining a faster algorithm. We describe the tool implementing this algorithm in section 5. Finally, some conclusions and the future work appear in the last section.

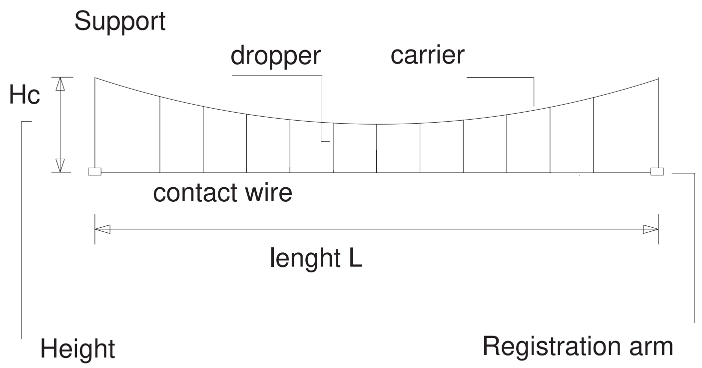

Figure 1: Model of Catenary

2 The catenary model

The conventional catenary electrification system is designed for heavy-traffic mainline operation and it is useful for train speeds well above 200 kph. For such high-speed operation an essentially constant contact force must be maintained between the overhead contact-wire and the locomotive's pantograph power-collecting apparatus.

As we have previously indicated, in this paper we will use a classical model of the catenary wire appearing in railways, so we consider the catenary composed from a reduced range of elements, such as carrier, droppers, contact wire and registration arms (see Figure 1). The droppers are supporting the contact wire in order to obtain a horizontal line. The interval between two registrations arms will be called the gap. Thus, the set of data we will use in the design of the algorithms will be the following:

- Tension at the compensating pulley (Tension at the left support)
- Weight by unit of length of the carrier p
- Weight by unit of length of the contact wire q
- Specific weight of the material of the dropper (normally copper)
- Area of the section of the dropper S
- Fix weight for some components of the dropper P_g
- Length of the gap L
- Difference of height
- Number of droppers n
- Position of droppers $\{x_i,\ i = 1 \ldots n\}$

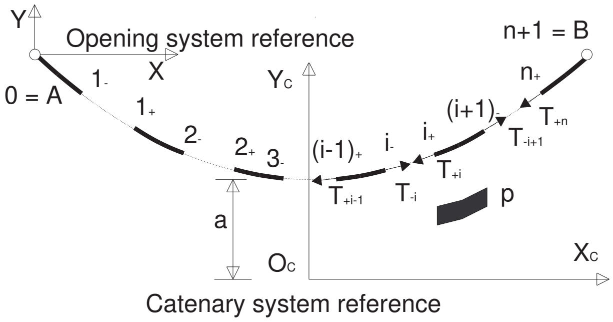

Figure 2: Representation of different strings of the catenary system

3 Algorithm 1: Calculation using the real weight of the carrier

To specify the length of the droppers, we need to know the deflection of the carrier in those points where the droppers will be put. Then, the carrier should be subjected to the uniform load of its own weight, say p, and the punctual loads of the droppers, $R_i, i = 1 \dots n$, where n is the total number of droppers. The load of a dropper is originated by its weight, which depends, among other factors, on its own length and the proportional part of the conductor it is supporting.

An important observation is that, as the loads operating on the carriers are vertical, we have as a consequence that the horizontal projection of the tension in the cable remains constant in every point, and then it is equal to the horizontal tension in the left point, T_{ax}.

The curve traced by the carrier is a catenary defined by a parameter a, depending on the load of the cable p, and the horizontal projection of the cable tension T_{ax}. Thus, as both variables remain constant, we may suppose that it is composed by a sequence of discontinues arcs from the same catenary, as it is showed in figure 2.

In that picture it is represented the catenary of a supportive cable in a system reference (catenary system). We may consider that every real point i is divided into two points (i^-) and (i^+), thus there are some segments of the real curve of the carrier, situated them between the generic points $(i+1)^-$ and i^-, some other segments not in the real curve, between the generic points i^- and i^+, representing the weight of this segment the load of the dropper at this point. And values T_i^- and T_i^+ represent respectively the left and right tension at point i, *i.e.* tension at points i^- and i^+.

If we consider the real gap, we may associate to it a system reference (gap system), with its origin at point A (left support of the catenary), then, the catenary system will vary for every segment of the real curve. So, the segment between real points $(i-1)$ and (i) will have as origin of its catenary

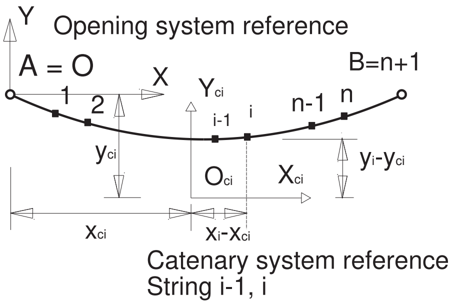

Figure 3: Catenary System, segment (i-1, i)

system $O_{ci}\,(x_{ci}, y_{ci})$. We have for point i with coordinates (x_i, y_i), following [1] and considering figure 2

$$y_i - y_{ci} = a \cdot \cosh\left(\frac{x_i - x_{ci}}{a}\right) \tag{1}$$

To calculate the catenary, our starting point, as it is showed in Fig. 3 is the set of data indicated in section 2.

We want to know the length of the droppers, (*i.e.* the full position $\{y_i,\ i = 1 \ldots n\}$ of the cables with reference to the gap system). Thus, if we suppose that each suspension cable supports half of the weight of the contact wire situated between two consecutive droppers, we have the load of a dropper as:

$$R_i = q \cdot \frac{x_i - x_{i-1}}{2} + q \cdot \frac{x_{i+1} - x_i}{2} + P_{pi} + P_g, \quad i = 1 \ldots n \tag{2}$$

Where P_{pi} represents the weight of the dropper, which depends of its length, and which is unknown in principle, beginning with a value of 0. Then, with the tension T_a we do the first estimation of the parameter "a", and T_{ax}, for that we suppose that the load p in the cable is uniformly distributed among its horizontal length. Thus, the moment of the effort in the droppers and the weight of the cable with respect to the left support in the arm is:

$$M_{PB} = \sum_{i=1}^{n} R_i \cdot (L - x_i) + p \cdot \frac{L^2}{2} \tag{3}$$

Considering the steady equation of moments of the carrier with respect to the left arm, and knowing the cable tension at this support, we have

$$T_{ay} \cdot L + T_{ax} \cdot h - M_{PB} = 0\ , \quad T_{ay} = \sqrt{T_a^2 - T_{ax}^2} \tag{4}$$

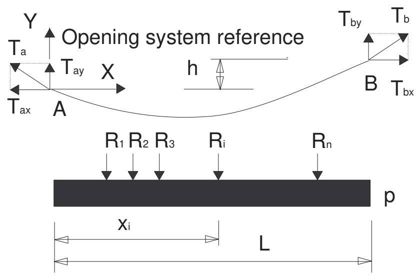

Figure 4: Diagram of free solid for the full carrier

From these equations, we obtain an equation depending on T_{ax}, which may be solved obtaining the parameter "a", as follows

$$
\begin{aligned}
T_{ax} &= \frac{2 \cdot M_{PB} + \sqrt{4 \cdot T_a^2 \cdot L^2 \cdot (L^2 + h^2) - 4 \cdot M_{PB}^2 \cdot L^2}}{p \cdot 2 \cdot (L^2 + h^2)} \\
a &= \frac{T_{ax}}{p} \qquad (5)
\end{aligned}
$$

Next, we calculate the positions of the different points of the droppers in the carrier, refereed with respect to the gap system. Initially we have:

$$x_0 = y_0 = 0 \ , \quad T_0^+ = T_a \qquad (6)$$

and going on for $i = 1 \ldots n+1$ we may calculate different parameters. Right tension at each point will allow us to calculate the position of the origin of the catenary system O_{ci} for the string $(i-1, i)$, and according with the wire static equations, we can calculate the position and tension at point i:

$$
\begin{aligned}
y_{ci} &= y_{i-1} - \frac{T_{i-1}^+}{p} \ , \quad i = 1 \ldots n+1 \\
x_{ci} &= x_{i-1} - a \cdot \operatorname{arccosh}\left(\frac{y_{i-1} - y_{ci}}{a}\right) \ , \quad i = 1 \ldots n+1 \qquad (7) \\
y_i &= y_{ci} + a \cdot \cosh\left(\frac{x_i - x_{ci}}{a}\right) \ , \quad i = 1 \ldots n+1 \qquad (8) \\
T_i^- &= p \cdot (y_i - y_{ci}) \ , \quad T_{iy}^- = \sqrt{(T_i^-)^2 - T_{ax}^2} \ , \quad i = 1 \ldots n+1 \qquad (9) \\
L_{pi} &= H_c + y_i \ , \quad P_{pi} = L_{pi} \cdot S \cdot \gamma \ , \quad i = 1 \ldots n \qquad (10)
\end{aligned}
$$

Which allows us to calculate R_i from the formula (2). Also, we obtain

$$T_{iy}^{+} = T_{iy}^{-} - b \cdot R_i, \; T_i^{+} = \sqrt{(T_{iy}^{+})^2 + T_{ax}^2}, \; i = 1 \ldots n \tag{11}$$

where $b = \begin{cases} +1 & \text{If } y_{i-1} > y_i \\ -1 & \text{If } y_{i-1} < y_i \end{cases}$

$$y_{ci+1} = p \cdot y_i - \frac{T_i^{+}}{p} \ldots, \; \textit{repeating the process from eq. 7} \tag{12}$$

The results thus obtained are conditioned by the value of parameter "a", estimated at the beginning of the process, which is approximated. A method to know a better value, closer to the real one, consists in introducing a tolerance error ϵ for the value y_{n+1} calculated in eqn. 8, when we know the exact value $y_b = y_{n+1} = h$. Then, if we calculate again the tension in B or in $(n+1)^{-}$, according with this new estimation:

$$T_{n+1}^{-} = p \cdot (h - y_{cn+1}) \tag{13}$$

And finally, the value of "a", when $|y_{n+1} - h| \geq \epsilon$ is obtained as follows:

$$a = \frac{T_{ax}}{p} = \frac{\sqrt{(T_{n+1}^{-})^2 - (T_{n+1y}^{-})^2}}{p} = \frac{\sqrt{[p \cdot (h - y_{cn+1})]^2 - (T_{n+1y}^{-})^2}}{p} \tag{14}$$

Where T_{n+1y}^{-} is given by eqn. 9. Then, we may continue the process obtaining a new value of "a" from eqn. 6 until the error of the tension in the position of B is under the value of ϵ.

4 Algorithm 2: Calculation using a uniform distributed load

We have another method to solve our problem, by taking as hypothesis that the weight of the carrier may be uniformly distributed among the horizontal length, in place of the distribution among the full length considered in the previous method. This hypothesis, although it is only an approximation, allows us to obtain results not much different with respect to those obtained with the previous (more exact) hypothesis, but we can obtain a faster algorithm.

The new method, consists in calculating at first place the weight of the dropper assuming initially a null value following equation 2, continuing with the calculation of T_{ax} and T_y as indicated in equations 4 and 5. After that, we calculate the deflection in the carrier, by applying equations of static according with Figure 5, where point C_i with $i = 1 \ldots n$, is considered as center of moments:

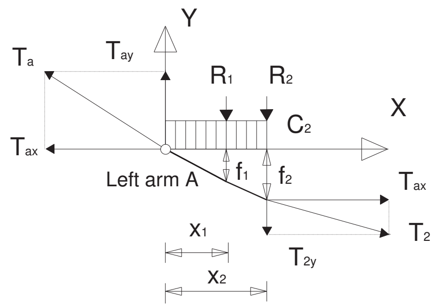

Figure 5: Diagram of free solid for a segment of carrier

$$\sum M_{C_i} = 0, \ \ f_i = \frac{1}{T_{ax}} \cdot [T_{ay} \cdot x_i - p \cdot \frac{x_i^2}{2} - \sum_{j=1} i R_j \cdot (x_i - x_j)] \ , \ \ i = 1 \ldots n \quad (15)$$

Once obtained the deflection of the carrier at the position of the droppers, we may calculate the length of the droppers and the weight of these according with equation 10, hence we calculate again the horizontal projection of the cable tension, according with equation 5. This new value is called T'_{ax}, while the previous one remains being T_{ax} if it is verified:

$$\left| \frac{T_{ax} - T'_{ax}}{T'_{ax}} \right| \leq \epsilon \quad (16)$$

This condition means that the previous value of T_{ax} is correct and our problem is solved. In the opposite case, it is necessary to obtain a more exact value, thus we make $T_x = T'_{ax}$ repeating the process until condition 16 is satisfied, thus obtaining the length of the droppers.

5 The software tool

The software tool, which we have named CALPE, has been developed following the second (faster) algorithm here presented. It has been written on an object-oriented database system with a visual interface under Windows 95. This framework is supported in the Visual FoxPro (© Microsoft) environment, and it is currently used by RENFE, the Spanish company of railways, in the development of its electrical catenary systems.

The tool, whose current user interfaces are in Spanish, consists of a menu, where we may choose several options, among them the maintenance of the database system, designed with several files implementing the different tables of a relational database system following a previously designed entity-relation scheme. These tables implement the different components of

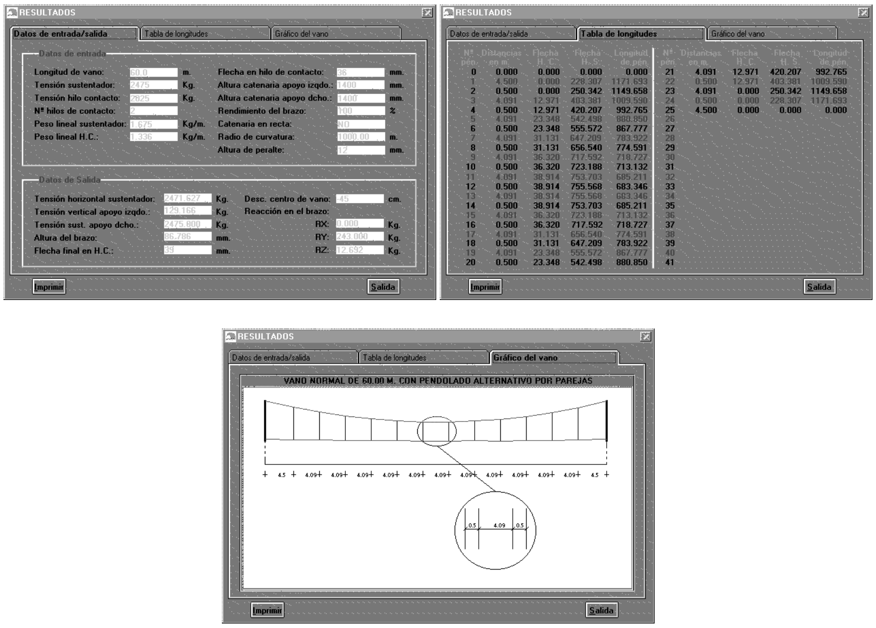

Figure 6: Some windows of the tool

our catenary model, that is different types of wires (carrier and contact), droppers, and so on.

The main procedure in the tool is the design of the catenary. This procedure has been called "*normal gap*", because this algorithm is the first step in the automatic design of different types of catenaries, such as Y-gap, sectioned gap, tangential gap and some others.

This procedure is also divided into two parts, defined each one of them over a window in a friendly user interface. The first one consists of the input of the different data types, selected among the previously introduced components in the database system, and some other new data types. After that, we have the output results presented in another window (see Fig. 6), again divided into three sections, one with the main input/output data, another with the dropper length table, and finally a picture of the catenary.

6 Conclusions and Future Work

We have presented two algorithms to calculate an electric power line (catenary) for railways. In the first one, we have considered the exact weight of the carrier, while in the second one we have considered a uniformly distributed weight along the horizontal length, obtaining similar results, but the second one being faster in the execution. This algorithm has been implemented in a software tool (CALPE), developed by the University of Castilla-La Mancha, and currently used by RENFE (spanish company of railways) to design the catenaries of its railways, obtaining better quality lines.

As future work, we will look into new considerations on the design of the catenary, such as the deformation of the catenary due to the up pressure of the pantograph, a phenomenon known as gap rigidness. This rigidness is a little lower in the center of the gap than in the supports, and this may cause oscillations in the pantograph. In order to avoid this problem, the contact wire may have a deflection in the center, however this deflection also causes a distending in the carrier due to the alteration in the initial distribution of the loads. Thus, another way to solve this problem is by using a more advanced solution known as the Y-dropper, consisting in placing a false carrier in the supports, obtaining a rigidness more uniform along the gap.

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