Abstract

Nowadays, society claims for better transport media, that mean, faster and more confortable. In a railway environment, faster transport can be seen as a problem of energy transmission. This energy transmission is produced in the interaction of the pantograph and catenary. An accurate model of this interaction becames essential due to the fact that a loss of contact between the pantograph and the catenary results in a loss of velocity. A very useful tool to study the behaviour of the model is the simulation by means of a set of algorithms, which usually needs a considerable amount of computation and memory storage. In this work, the calculation of the dynamical equation of the pantograph-catenary interaction in a stitched span is considered, not only from a mechanical point of view (an accurate model of the system), but also from the simulation point of view by means of a High Performance Computing Algorithm. By using this HPC approach, the requirements of memory storage and the execution time needs to solve the dynamical problem have been dramatically reduced.

Keywords: High performance computing, interaction pantograph/catenary, dynamic equation, sparse linear algebra libraries.

1 Introduction

The evolution in the transport market resulting from the globalization of the economy, the increasing deregulation of the markets, the competition on the base of a customer service, the growing environmental concerns, and the need to ensure a long term operational profitability consistent with prevailing economic reality are driving to a radical change (structural and cultural) in the railway sector, centered on innovative approaches to business and services, conductive to a future-oriented and market-driven posture in the global transportation marketplace.

The survival in the ever evolving and highly competitive transport market, implies a continuous search for the roots of excellence enabling railway operators and supplying industry to achieve a world-class profile. This quest for excellence has to cover the entire range of business activities, beginning with market demand and ending with customer satisfaction. It will entail the need for an integrated development and timely deployment of the adequate organizational, technological and skill infrastructures.

Thus, the fulfillment of this strategy needs new construction concepts, among them, the development of more performant coupled pantograph/ catenary systems and a wide utilization of new technologies.

Furthermore, the multitude of electrification systems currently in use throughout Europe and subsequently the massive capital investment that would be necessary in order to implement any sort of European wide harmonised solution, preclude to envisage any major change in this field in the foreseeable future. A detailed study of electrification systems and catenary/pantograph technology considering economic aspects may be found in [4].

During the recent last years, passengers transportation by railway has experienced a considerable increase in some European countries (Germany, France, Spain, ...). For that reason, reaching of higher velocities in railways has become a very important target. In that scenario, the pantograph/catenary system, with its dynamic behaviour, becomes a crucial component (see [5, 2, 3]), because at high speed it is very difficult to guarantee the permanent contact of the pantograph head and contact the wire, more over without the increasing of noise and wear.

In order to achieve adequate benefits in circulation of railway units, the panthograph-catenary contact force has to be mantained as uniform as possible, avoiding contact loses. By developing a mathematical model, that allows us to simulate the mechanic behaviour of the system, can be helpful in specifying optimal assembly conditions in the catenary or aereal contact line.

The numeric integration of the differential equation of the pantograph-catenary system presents a great difficulty due to the high number of variables that appear in the mathematic formulation of the equation, which has the consecuents requeriments of memory and computation time. The aereal contact line is built considering a range of spans, normally 15 or 20, which has $60m$ long, constituting each series an independent system, being necessary to consider a lot of short cable elements, when carrying out discretization with Finite Elements Method.

Besides, the resulting differencial equations are not lineal, when varying the system's stiffness matrix each time, being also a sparse matrix type. In order to solve these difficulties, in the present work a method is presented to be able to integrate numerically and with a computacional efficient form the differential equation of the pantograph-catenary system. This work has been structurated as follows:

In section 2 the dynamic equations are displayed. In sections 3 to 7 the composition and structure of the different elements of this equation is explained, section 8 explains the integrated numerical algorithm, which is based in the central difference method. Sections 9 and 10 introduce computacional aspects. Section 11 presents the cases of study carried out and the experimental results obtained. Finally, the conclusions and future work are outlined.

2 Dynamic equations of the pantograph-catenary system

The dynamic equations of the pantograph-catenary system particularized for an instant of time t_n are:

$$\begin{pmatrix} M & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} \ddot{Q}_n \\ \ddot{\lambda}_n \end{pmatrix}+\begin{pmatrix} C & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} \dot{Q}_n \\ \dot{\lambda}_n \end{pmatrix}+\begin{pmatrix} K_n & A_n^T \\ A_n & 0 \end{pmatrix}\begin{pmatrix} Q_n \\ \lambda_n \end{pmatrix}=\begin{pmatrix} R_n \\ 0 \end{pmatrix} \tag{1}$$

Where M is the mass matrix and C is the damping matrix, considering both constant. The other terms and matrices that appear in the equation, will vary for each instant of time t_n, being K the stiffness matrix, A the restriction condition submatrix, R the independent term vector, Q and λ are the generalized coordinates and restrictions forces vectors, respectively. Each matrix can be separated into 2 submatrices corresponding catenary and pantograph. this can be aplicable to independent term and generalized coordinated vectors too.

To obtain the different elements of the equation, catenary and pantograph have to be considered independently. Talking about catenary, semidiscretization by means of the Finite Elements Method (FEM) will be used, whereas with regards to the pantograph the model which is suministrated by the manufacturer formed by a set of masses, springs and shock absorbers will be used.

Modelizing the catenary, it must be considered the different types of elements conforming it, according to Figure 1: carrier, droppers and contact wire. At the same time, we need to consider the effect of the compensation arm.

In order to modelize the carrier and contact wire, the Euler-Bernuilli differential equation for a pretensed beam has been considered:

$$\frac{p}{g}\ddot{y} = -EIy^{IV} + T_x y'' - p, \tag{2}$$

where p represents the lineal weight of the beam, T_x the horizontal component of the mechanical tension, E the elastic module, g gravity acceleration and I the diametral moment of inertia of the beam's section.

This equation allows to carry out semidiscretization with FEM, using either Ritz's method or Galerkin's weigthed residual [10]. Despising dynamic effects, the static balanced equation for a beam's segment is obtained as follows:

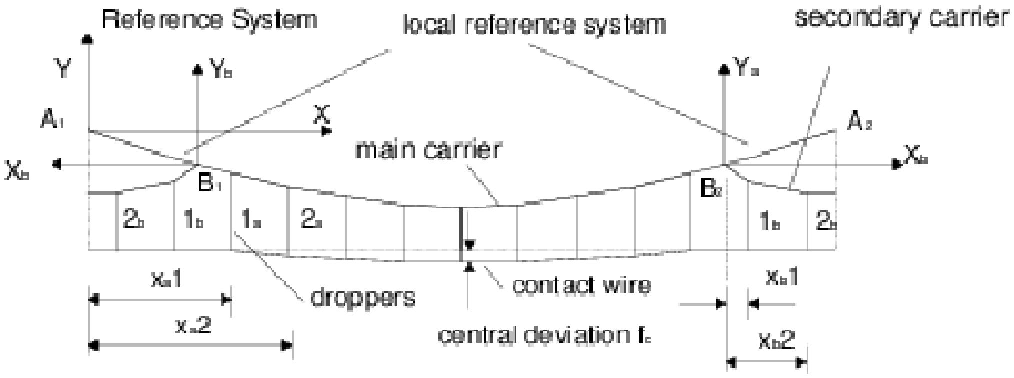

Figure 1: A span of a stitched catenary

$$kq = r, \tag{3}$$

In this case, each node has 2 generalized coordinates, the position given by the coordinate y, and the turn angle Θ_i, having for the independent term and the generalized coordinates:

$$r = \begin{bmatrix} -\frac{pl}{2} \\ -\frac{pl^2}{l2} \\ -\frac{pl}{2} \\ \frac{pl^2}{l2} \end{bmatrix}, \quad q = \begin{bmatrix} y_i \\ \theta_j \\ y_i \\ \theta_j \end{bmatrix}, \tag{4}$$

and for the element's stiffness matrix:

$$k = \frac{EI}{l^3} \begin{bmatrix} 12 & 6l & -12 & 6l \\ 6l & 4l^2 & -6l & 2l^2 \\ -12 & -6l & 12 & -6l \\ 6l & 2l^2 & -6l & 4l^2 \end{bmatrix} + \frac{T_x}{30l} \begin{bmatrix} 36 & 3l & -36 & 3l \\ 3l & 4l^2 & -3l & -l^2 \\ -36 & -3l & 36 & -3l \\ 3l & -l^2 & -3l & 4l^2 \end{bmatrix}. \tag{5}$$

Despising the bending stiffness, previous expresions can be simplified obtaining the next terms:

$$k = \frac{T_x}{l} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}, \quad r = \begin{bmatrix} -\frac{pl}{2} \\ -\frac{pl}{2} \end{bmatrix}, \quad q = \begin{bmatrix} y_i \\ y_j \end{bmatrix}. \tag{6}$$

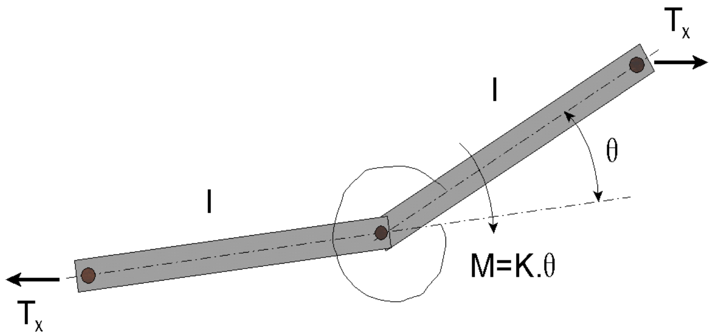

Figure 2: Model of a spring with torsional spring

In this case, there is only one coordinate associated to the node, which is related to its vertical position. This equation correspond to the pretensed wire model which is equivalent to consider the cable as a serial of elements either stiffs or rods joint by joints without bending stiffness.

However, doing a simple change in the previous equations, the bending stiffness can be considered. This change consist in mantaining one generalized coordinate per node, introducing a stiffness torsional spring K in the nodes, as Figure 2 shows. In this case the element's stiffness matrix shows as follows:

$$k = \frac{T_x}{l}\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} + \frac{k_t}{l^2}\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}. \tag{7}$$

In this case, the contact wire has been modelized as a pretensed beam while the carrier has been considered as a pretensed wire with torsion spring.

3 Modelizing droppers

Droppers behave as elastic bars, which deform themselves from an initial length, maintaining a generalized coordinate per node, corresponding to ordinate or vertical movement. The stiffness matrix, the independent term and the coordinate vector will be as follows:

$$k = \frac{EA}{l}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}, \quad r = \begin{bmatrix} \frac{EAl_0}{l} \\ -\frac{EAl_0}{l} - P \end{bmatrix}, \quad q = \begin{bmatrix} y_i \\ y_j \end{bmatrix}, \tag{8}$$

being A the straight section of the dropper, l the assembly length, l_o its initial length (slightly different from the assembly length) and P the weight of the dropper. There is a relation between the assembly length l, initial length l_o and preloaded assembly F:

$$l_0 = l(1 - \frac{F}{EA}). \tag{9}$$

Then, the independent term of the equilibrium equation can be expressed as:

$$r = \begin{bmatrix} EA - F \\ -EA + F - P \end{bmatrix}. \tag{10}$$

The preloaded value F has to be determined by static analysis being fundamental for the calculation of the dropper's length l, denpending on their own weight and the contact wire's weight that supports them. Determining the dropper's assembly length is fundamental for the adecuate instalation of the catenary.

Finally, taking into account that droppers cannot work with compression and that in the dynamic interaction, the length of the dropper can vary with the difference between the ordenates in the extreme nodes. When the length of the dropper defined that way, is shorter than initial length l_o, it does not work and we don't have to take it into account when ensamblying the stiffness matrix of the system. The condition to take into account the conexion effect in the droppers is:

$$y_i - y_j \geq l_0 = l(1 - \frac{F}{EA}). \tag{11}$$

According to this, the stiffness of the catenary matrix and independent term will not be constant but they have to be modified taking into account active droppers.

4 Modelizing registration arm

Registration arm can be considered as a spring that exert a vertical force over f_b contact wire, given by the equation:

$$f_b = f_0 + (y_h - y_A)k_b, \tag{12}$$

Where k_b is the apparent stiffness of the arm, y_A the dinamic position of the holding point, y_h the static position of the holding point which is already known and f_0 is the contact wire weight which supports the arm for the assembly static position.

Modelizing this previous elements, the stiffness matrix of the catenary matrix and independent term can be assemblied.

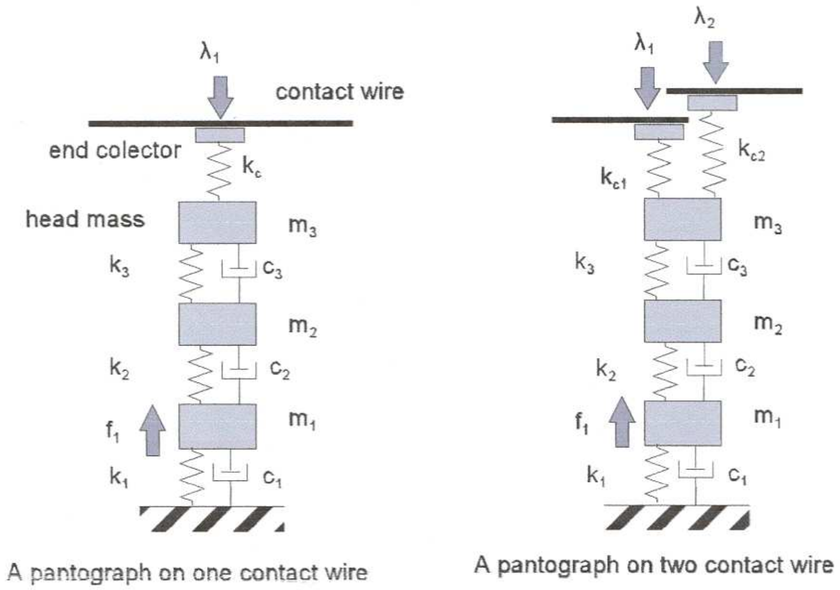

Figure 3: Modelization of the interaction between the pantograph and one contact wire (a) and two contact wire (b)

5 Mass of the catenary and damping matrix

The mass matrix of the catenary matrix can be determined by the described procedures using FEM. Two types of matrices can be considered: consistent mass matrix and diagonal mass matrix. In this case it has been considered the second one.

For the damping matrix of the catenary damping matrix it has been considered a Rayleigh damping, being a lineal combination of mass and stiffness matrices. In this case stiffness matrix is considered constant with all droppers working.

6 Modelizing dynamic of the pantograph

The pantograph is modelized as a set of joined masses by springs and shock absorbers, as it is shown in Figure 3, which values are specified by the manufacturer. To facilitate the diferencial equation integration, an aditional element without any mass has been added, the terminal colector, which is grazing the contact wire and it is joined to the head mass by a rigid spring k_c. In case that the pantograph interacts with two wires, two colectors are needed to be considered, as Figure 3 shows. With this figure it is easy to configure mass, stiffness and damping matrices as well as the independent term of the pantograph.

It is easy to see that the mass matrix of the pantograph is diagonal and constant. The stiffness mass of the Pantograph is not always constant but it can vary according to the relative distances between masses. The same thing happens to independent term. This term include the different forces acting on the masses, like friction forces, whose orientation varies according to the direction of the relative speed between masses.

7 Constraints conditions

Establishing the system differential equations, the constraint conditions of the pantograph-catenary system has to be considered. These constraints affects to the pantograph terminal colectors and the contact wire nodes surrounding pantograph position. In order to be able to integrate numerically the differential equation, it is convenient that the contact is distributed but not puntual, according to a determinated law or weighted function (see [3]). In this case it has been obtained good results using a weighted function of hermite type. Vertical position y_c of the pantograph's terminal colectors will be calculated from:

$$\int_a^b f(x)(y(x) - y_c)dx = 0 \rightarrow A_n Q_n = 0, \tag{13}$$

where $y(x)$ represents vertical position of the contact wire, a and b define horizontal position of terminal colector for each considered instant and $f(x)$ represents ponderation or distribution function. The previous equation shows that vertical position y_c of the pantograph's terminal colector is determined by the position of the nodes of the contact wire situated around it. Once the integral is solved, the previous equation can be expressed as the product of the constraint conditions matrix for the considerated instant A_n and the generalized coordinate vector Q_n from where vertical position y_c of terminal colector can be obtained.

8 Numerical integration using central difference method

In order to simplify, the following study assumes one catenary with one contact wire and therefore just one pantograph with one terminal colector. In order to come to the numerical integration of this equation by the central difference method in a more efficient way, having into account that the damping matrix C is not diagonal, the initial equation has been sligthly modified, correcting half step of time in the speed vector $\dot{Q}_n$ changing this by $\dot{Q}_{n-\frac{1}{2}}$. According to [10], the resulting error of this integration is despisable when damping values are small, as in this case. This allows to integrate with simple matricial operations, without having to solve a linear equation system for each integration step:

$$\begin{pmatrix} M & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \ddot{Q}_n \\ \ddot{\lambda}_n \end{pmatrix} + \begin{pmatrix} C & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \dot{Q}_{n-\frac{1}{2}} \\ \dot{\lambda}_{n-\frac{1}{2}} \end{pmatrix} + \begin{pmatrix} K_n & A_n^T \\ A_n & 0 \end{pmatrix} \begin{pmatrix} Q_n \\ \lambda_n \end{pmatrix} = \begin{pmatrix} R_n \\ 0 \end{pmatrix} \tag{14}$$

by approaching the velocity by:

$$\dot{Q}_{n-\frac{1}{2}} = \frac{1}{\Delta t}(Q_n - Q_{n-1}), \tag{15}$$

and acceleration:

$$\ddot{Q}_n = \frac{1}{\Delta t^2}(Q_{n+1} - 2Q_n + Q_{n-1}), \tag{16}$$

Replacing the results in (15) and (16) in differential equation:

$$\begin{aligned}
\frac{1}{(\Delta t)^2}\begin{pmatrix} M & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} Q_{n+1} \\ \lambda_{n+1} \end{pmatrix} &= \begin{pmatrix} R_n \\ 0 \end{pmatrix} - \begin{pmatrix} K_n & A_n^T \\ A_n & 0 \end{pmatrix}\begin{pmatrix} Q_n \\ \lambda_n \end{pmatrix} - \\
&+ \frac{1}{(\Delta t)^2}\begin{pmatrix} M & 0 \\ 0 & 0 \end{pmatrix}\left(\begin{pmatrix} Q_n \\ \lambda_n \end{pmatrix} + \Delta t\begin{pmatrix} \dot{Q}_{n-\frac{1}{2}} \\ \dot{\lambda}_{n-\frac{1}{2}} \end{pmatrix}\right) - \\
&- \begin{pmatrix} C & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} \dot{Q}_{n-\frac{1}{2}} \\ \dot{\lambda}_{n-\frac{1}{2}} \end{pmatrix}
\end{aligned} \tag{17}$$

being M a diagonal mass matrix, the previous equation can be considered as an operator that allow us to obtain coordinate vector Q_{n+1} in instant t_{n+1} with simple matricial operations, directly.

However, constraint conditions prevent to determine the restriction forces term λ_{n+1}, as well as the terminal collector's position $y_{c_{n+1}}$, since mass matrix presents the corresponding zero terms. To solve this problem, the terminal collector's position is determinated by the equation (13), then if $y_{h_{n+1}}$ represents the pantograph's head mass position, which has been determinated previously by (17), the restriction force may be:

$$\lambda_{n+1} = k_c\left(y_{c_{n+1}} - y_{h_{n+1}}\right). \tag{18}$$

This completes the integration cycle. According to that, the algorithm is structurated as follows:

1. Insert line's assembly data. Form the stiffness matrix and catenary's independent term. Add pantograph's stiffness matrix and independent term, forming initial stiffness matrix K_1. Initially we will consider that every dropper works.

2. Form catenary's mass matrix and add pantograph's mass matrix forming mass matrix M of the pantograph-catenary system, obtaining a diagonal matrix.

3. Form catenary's damping matrix as a lineal combination of stiffness and masses matrixes, supposing a Rayleigh damping type, next pantograph's damping matrix is added, forming system's damping matrix C.

4. Form constrain condition matrix in the initial instant A_1 by equation (13) assemblying it with initial stiffness matrix K_1.
5. Determine generalized coordinate vector and retriction forces Q_1 and λ_1 for initial instant solving this lineal system:

$$\begin{pmatrix} K_1 & A_1^T \\ A_1 & 0 \end{pmatrix} \begin{pmatrix} Q_1 \\ \lambda_1 \end{pmatrix} = \begin{pmatrix} R_1 \\ 0 \end{pmatrix}. \tag{19}$$

   In fact, this step consists on solving the static problem in the pantograph-catenary interaction.
6. Establish initial conditions for speed on generalized coordinates $\dot{Q}_{\frac{1}{2}} = 0$
   (a) Integration cycle for differential equations in time
   (b) Modify step cycle $(n+1 \to n)$, particularly generalized coordinate and speed vector has to be reconverted in the generalized coordinates

$$Q_{n+1} \to Q_n, \;\; Q_{n-\frac{1}{2}} \to Q_{n+\frac{1}{2}},$$

   as well as stiffness matrix and independent term

$$K_{n+1} \to K_n, \;\; R_{n+1} \to R_n,$$

   Initial position corresponding instant 1 to instant 0
   (c) Calculate catenary's and pantograph's generalized coordinates corresponding instant $n+1$, Q_{n+1} according to equation (17)
   (d) Form matrix corresponding to constrain conditions for instant $n+1$, A_{n+1} by equation (13). Determine GC asociated to the colector, $y_{c_{n+1}}$.
   (e) Determine contact force (or forces), λ_{n+1} according to equation (18).
   (f) Update system's stiffness matrix, corresponding instant $n+1$, K_{n+1}, and independent term $R_{n+1,}$ taking into account:
      - Desconection effect on droppers
      - Resort's stiffness variation associated to pantograph's masses, according to relative positions of these masses.
      - Pantograph's friction forces. According to relative speed between masses, this forces present one or other direction.
      - Pantograph's contact lose effect. In case that this will take place, stiffness associated to terminal collector is cancelled.
   (g) Update constrain conditions of submatrix A_{n+1} assemblying it in stiffness matrix K_{n+1}.
   (h) New value of speed for instant $n+\frac{1}{2}$: $Q_{n+\frac{1}{2}}$ by equation (15).
   (i) End of time's cycle returning at point (6b)

9 High Performance Computing Approach

In order to solve a mathematical problem in a efficient way on a computer, the following steps are involved [13]

1. Making a mathematical model of the problem, translating the problem into a mathematical language, eg. ordinary differential equations.

2. Finding or developing constructive methods for solving the mathematical model, that is, a literature search to find what methods are available for the problem.

3. Identifying the best method from a numerical point of view.

4. Implementing on the computer the numerically effective method identified in the previous step.

In general, the developed software has to be a *high-quality mathematical software* which guarantees a good solution to the problem. This high quality mathematical software should have the following features

1. Power and flexibility: The developed algorithms can be used to solve several different variations of the original problem and the closely associated problems.

2. Easily read and modified: The software should be well documented. Then, the documentation should be clear and easy to read, even for nontechnical users.

3. Portability: The implemented algorithms should be able to run on different computers with few or no changes.

4. Robustness: It should be able to deal with unexpected situations during execution.

5. Based on a numerically effective algorithm: It should be based on an algorithm that has attractive numerical properties.

In the particular case of a matricial algorithm, as our case, it is numerically effective if has the following features:

1. General-Purpose: The algorithm should work for a wide class of matrices.

2. Reliable: The algorithm should give warning whenever it is on the verge of breakdown due to excessive roundoff errors or not being able to meet some specified criterion of convergence.

3. Stable: Total rounding errors of the algorithm should not exceed the errors that are inherent in the original problem.

4. Efficient: The efficiency of an algorithm is measured by the amount of computer time consumed in its implementation. Theorically, the number of floating point (**flop**) operations needed to implement the algorithm indicates its efficiency.

5. Economic in use of storage: Usually, about n^2 storage location are required to store a dense matrix of order n. If an algorithm requires storage for several matrices during its execution, a large number of storage locations will be needed even if n is moderate. Thus, it is important to give special attention to economy of storage while designing an algorithm. By carefully rearranging an algorithm, it is possible to greatly reduce its storage requeriments studing the special structure of the problem or avoiding to generate a matrix during execution of the algorithm that it is not needed for the future use, it should be overwritten by another computer element.

The two last points are specially important in the problem solved in this work.

The enhancements carried out in this work has been related to the way of storing the matrices involve in the resolution of the dynamic equation, as well as the use of suitable methods for solving the associated static problem.

In particular, the sparsity and simmetry of the stiffness matrix has been exploted, improving the efficiency of the implementation and dramatically reducing the memory storage requirements, and making use of iterative methods based on Krylov projection methods, the execution time spend for solving the static problem has been also dramatically reduced.

Therefore, a High Performance Implementation has to take into account the features of current architectures like, for example, cache memory. These features are particularly important when rebuilding the traditional algorithms to a block-oriented implementations. Block-oriented algorithms reduce drastically the data flow between main memory and secondary memory enhancing the performance of the final implementation. These good features have been obtained by using standard libraries in the implementations.

10 Linear Algebra Libraries

One of the keys to make an efficient use of the current computers is to avoid unnecesary reference to main memory. It is well known that data flows from main memory to registers and viceversa. This flow of data could be more expensive than the arithmetic operation over these data. According to that situation, the design of new algorithms must consider the minimization of data movements. With this goal in mind, the algorithms developed in the present work have been implemented taking into account the following features:

- Portability: To achieve portable algorithms Standard Linear Algebra Libraries, such as BLAS (I, II and III) and SPARSKIT, have been used.

- Efficiency: Good performance has been achieved by using block oriented algorithms, exploiting the memory hierarchy of the current computers.

The BLAS [11](*Basic Linear Algebra Subroutines*) library includes subroutines for common linear computations such as dot-products (BLAS-I), matrix-vector multiplication (BLAS-II), and matrix-matrix multiplication (BLAS-III).

Sparse matrices appear on a lot of current problems in science and engineering. Due to that fact, an intensive research is being carried out in this area producing lot of storage schemes and methods to deal with sparse matrices. SPARSKIT [12] is a software packet which allows us to work with different storage schemes (COO, CSR, CSC, etc) and iterative methods for solving sparse systems of equations. This packet is divided into several modules for conversion of storage scheme (*FORMAT* module), basic linear algebra operations over sparse matrices (*BLASSM* and *MATVEC* module), system of equations solvers (*ITSOL* module), etc.

11 Experimental Results

In this section, the experimental results obtained with the new HPC implementation of the algorithm for solving the dynamic problem is presented, considering a stichted catenary with one contact wire.

The test battery used in the experiments is shown in Table 1. As the number of spans has been settled to 10 and the lenght of the span to 60 meters, this test battery is considered as realistic covering 600 meters of simulation. This amoung of distance is enought to obtain real results. One parameter to take into account is the number of pantographs. This parameter varies between 1 to 4.

Test	nv	np	lv	npan
1	10	9	60	1
2	10	9	60	2
3	10	9	60	3
4	10	9	60	4

Table 1: Test Battery

Where nv is the number of spans, np is the number of droppers, lv is the length of the section and $npan$ the number of pantographs.

The experiments have been carried out in a Pentium III-650MHz with Red Hat Linux V. 9 operating system. The experimental platform has 385 MBytes of RAM memory and a cache memory of 256 KBytes. BLAS and SPARSKIT libraries used in this work have been compiled in this machine in order to obtain better performances.

Table 2 shows the spent in the memory storage requiriments between storages the stiffness matrix in dense format and in sparse format. Then, the current storage requirement is $(N \times N) \times 8$ bytes, being N the leading dimension of stiffness matrix and

8 the number of bytes for double data type. On the other hand, nz means non-zero elements. In this case, the storage requirements is $(nz \times 2) \times 8$. This reduction in terms of memory requeriments benefits the efficiency of the algorithms.

Test	dense format (bytes)	sparse format (bytes)	% Reduction of memory
1	183322952	377072	99.7943
2	183936200	377648	99.7947
3	184550472	378224	99.7951
4	185165768	378800	99.7954

Table 2: Reduction of memory requirements

The execution time has also been drastically reduced with respect to the original implementation. In table 3 the execution time of the previous test is summarized.

Test	Execution time (secs)
1	1586.4384
2	2452.3221
3	3363.9500
4	4089.8153

Table 3: Execution time in seconds

Figure 4 shows simulation results for test number 1. In order to clarify the representation of the results, details of the heights of contact wire and the pantograph forces are plotted.

12 Conclusions and Future Work

In this work a High Performance Computing Algorithm has been developed for solving the static equilibrium equation of the pantograph/catenary system of High Speed Railways. This new approach is based on the implementation philosophy of High Quality Sotfware.

The experimental results show that the HPC resulting algorithm provides spectacular reduction in the memory requirements as well as in execution time.

By using BLAS and SPARSKIT standard linear algebra libraries, two secondary objectives, but not less important, are achieved, i.e., portability and efficiency.

This work is the initial point of a lot of computational efforts in order to apply the HPC philosophy to different algorithms developed by the authors [17] following tradicional implementations. So, the future work could be outlined in the following points:

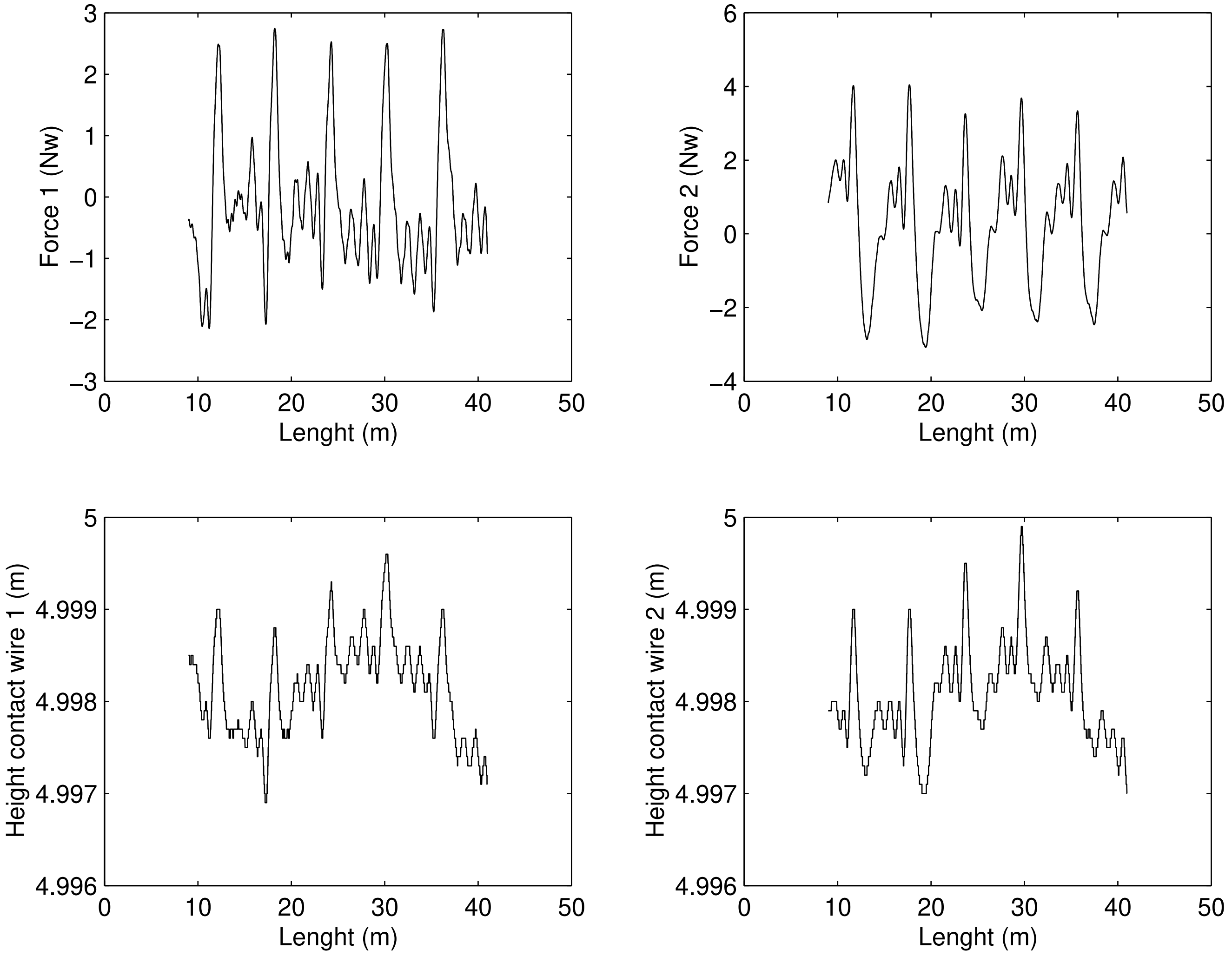

Figure 4: Results obtained for case of study 1

- To consider the solution when the spans contains different number of droppers and these are not equidistant.
- According to the results obtained for the dynamical problem, thinking about parallel implementations on shared memory platforms based on threads [14] or on distributed memory platforms based on MPI [15] and using the standard library PETSC [16], overall for solving the system of equations associated to the solution of the static problem.

The obtained algorithms will be used by RENFE, the Spanish Railway Company, in the design of high speed railways (AVE program).

13 Acknowledgments

We will thank Jesús Montesinos, engineer of RENFE, for its technical support and good knowledge of the real problem beyond the simulation world.

References

[1] Poetsch G., J. Evans, R. Meisinger, W. Kortum, M. Baldauf, A. Veitl and J. Wallaschek. *"Pantograph/catenary dynamics and control"*, Vehicle System Dynamics, 28:159-195, 1997.

[2] Poetsch G. and J. Wallaschek. *"Symulating the dynamic behaviour of electrical lines for high-speed trains on parallel computers"*, International Symposium on Cable Dynamics, Lige, 1993.

[3] Simeon, B. and Arnold M. *"The simulation of pantograph and catenary: a PDAE approach"*, Technical Report 1990, Fachbereich Mathematik Technische Universitat Darmstadt, 1998.

[4] Garfinkle, M. *"Tracking pantograph for branchline electrification"*, Technical Report -, School of Textiles and Materials Technology, University of Philadelphia, 1998.

[5] Poetsch G., J. Evans, R. Meisinger, W. Kortum, M. Baldauf, A. Veitl, & J. Wallaschek.*"Pantograph/catenary dynamics and control"*, Vehicle System Dynamics, 28:159–195, 1997.

[6] Carsten, N. J. *"Nonlinear systems with discrete and continuous elements"*, PhD thesis, University of, 1997.

[7] K. J. Bathe. *"Finite Element Procedures in Engineering Analysis"*, Ed. Prentice-Hall, 1996.

[8] Thomas J. R. Hughes. *"The Finite Element Method"*, Ed. Prentice-Hall, 1987.

[9] Zienkiewics, R. L. Taylor. *"El Metodo de los Elementos Finitos"*, Ed. McGraw Hill, 1980.

[10] Cook D. C., Malkus D. S., Plesha M. E. *"Concepts and Applications of Finite Element Analysis"*, Ed. John Wiley and Sons, 1989.

[11] J. J. Dongarra, J. Du Croz, I. S. Duff and S. Hammarling. *"A set of Level 3 Basic Linear Algebra Subprograms"*, ACM Trans. Math. Soft, 1990.

[12] Yousef Saad. *"SPARSKIT: a basic tool kit for sparse matrix computations"*, University of Illinois and NASA Ames Research Center, 1994.

[13] B. Nath Datta, *"Numerical Linear Algebra and Applications"*, Broks/Cole Publishing Company, 1995.

[14] Mueller, F. *"Pthreads Library Interface"*, Institut fur Informatik, March, 1999.

[15] Gropp, W. and Lusk, E. and Skjellum, A. *"Using MPI: Portable Parallel Programming with the Message-Passing Interface"*, MIT Press, 1994.

[16] Batish Balay, William Group, Lois Curfman McInnes, B. Innes. *"PETSC 2.0 User's Manual"*, Mathematic and Computer Science Division, 2000.

[17] J. Benet, F. Cuartero and T. Rojo. *"A tool to calculate catenaries in railways"*, Seventh International Conference on Computer in Railways, COMPRAIL-2000, 2000.