# Advanced algorithm to calculate mechanical forces on a catenary

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## Abstract

In this work, an advanced algorithm for the mechanical calculation of the catenary system in a railway is presented. The main objective is to allow an increment in the speed in railways, by developing an accurate mechanical calculation of the catenary. Among the different types of these kind of electrical systems, the so called stitched catenary presents a better dynamical behaviour, because it maintains a more uniform stiffness along the span. Of course, the price to be paid consists on a greater complexity due to the existence of two kinds of carriers, the main and the secondary ones. Finally, the presented algorithm will be used to implement a software tool.
*Keywords: mechanical forces, stitched catenary, droppers lentgh*

## 1 Introduction

In order to obtain an adequate behaviour in the pantograph/catenary system, it is necessary the existence of adequate conditions in the line, and this requires, among other aspects, a very precise mechanical calculation. Recent investigations have focused on dynamical behaviour by dynamical simulations in order to allow a better interaction of the pantograph and the catenary [9, 5]. This paper follows a more traditional approach, focusing

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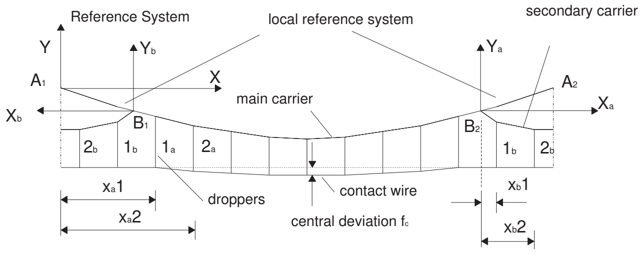



Figure 1: Model of stitched catenary

in the catenary, modeled, as usual, by a set of coupled strings. However, a different approach of the model of the catenary will be considered.

Of course, the best conditions in which the pantograph would obtain electric energy from the line are when the contact wire is parallel to the ground, and then, an important problem is to determine the exact length of the droppers in order to allow the contact wire to acquire the correct shape. So, the objective of this work is the development of a technique which allows us to implement a high precision calculation algorithm, and thus to develop a software tool to design high quality catenaries.

A first approach in that direction has been presented in [2], considering a normal curve to model the catenary. In this paper a more advanced model as the stitched catenary is considered, which contains four kind of wires: two carriers, the droppers and the contact wire (see fig. 1).

Thus, to obtain the better conditions to supply energy to the pantograph, it is necessary that the contact wire configures an adequate curve, that is, a parabolic arc with a deviation in the centre to compensate the difference of stiffness between the centre and the supports.

The paper is structured as follows. In section 2 the characteristics of the advanced model under study is described. In section 3 presents the algorithm for the calculation of forces, lengths and deviations. After that, in section 4 some of the features of the software tool implementing this algorithm is outlined. Finally, some conclusions and the future work appear in the last section.

## 2 The stitched catenary model

According with figure 1, there exists some droppers depending of the main carrier, and other ones depending on both secondary carriers. So that, three sets of droppers can be considered. The first one depending of the main carrier, and the second and third ones, with similar characteristics, depending of the secondary carriers left and right respectively.

Each set have associated a system reference and a numbering system. The main system reference have the origin in the left support $A_1$. This system is associated with first set of droppers, while the second and the third sets have associated their respective system reference, whose origin is situated in the union of the carriers $B_1$ and $B_2$.

The numbering of droppers depending of the main carrier is increasing from left to right, starting from the dropper situated in the union $B_1$, and finishing in $B_2$. For droppers depending of the secondary carriers, numbering is increasing from the first one in each case. The number of droppers and positioning will be similar for the two cases. Finally, the set of data used in the design of the algorithms will be the following:

- Number of droppers depending of the main carrier $n_a$
- Position of these droppers, refereed to its system reference $\{x_{i^a},\ i = 1 \dots n_a\}$
- Number of droppers depending of each secondary carrier $n_b$
- Position of these droppers, refereed to its system reference $\{x_{i^b},\ i = 1 \dots n_b\}$
- Length of the span $L$ (From $A_1$ to $A_2$)
- Distance from unions $B_1$ to the support $A_1$: $L_b$
- Height of catenary in the supports (distance from the carrier to the contact wire): $H_{c1}, H_{c2}, \dots,$
- Weight by unit of length of the carriers, $p_a$ to the main and $p_b$ to the secondary one
- Weight by unit of length of the contact wire $q$
- Specific weight of the material of the dropper (normally copper), and area of the section of the dropper $S$
- Fix weight for some components of the dropper $P_g$
- Tension at the compensating pulley (Tension at the left support) $T_1$
- Tension at the center of the secondary carrier $B_x$
- Tension at the contact wire $T_c$
- Deviation at the contact wire $f_c$
- Number of contact wires $n_c$
- Weight of the union $P_e$
- Weight of the secondary carrier $P_c$

## 3 Algorithm

This section describes the algorithm designed to calculate the forces, and some other necessary results in the model of catenary described in the previous section. A general approach to the algorithm is presented in table 1, and in the sequel the different steps followed for it are described.

Table 1: General Algorithm

| | | |
|---|---|---|
| 1. | data input | |
| 2. | Initialization | |
| | 2.1 | Own weight of droppers is null. $P_{pi} = 0$ |
| | 2.2 | Vertical distance from $B_1$ to $B_2$, $C = 0$ |
| 3. | **while** not convergence **do** | |
| | 3.1 | Calculation of vertical reaction at the unions $B_{1y}$ and $B_{2y}$ |
| | 3.2 | Calculation of reaction at the supports $A_x$, $A_{1y}$ and $A_{2y}$ |
| | 3.3 | Calculation of the length and deviation of the droppers of main carrier |
| | 3.4 | Calculation of the length and deviation of the droppers of secondary left and right carriers |

### 3.1 Calculation of the loads at the droppers of the secondary carrier

Unions $B_1$ and $B_2$ are the support for the secondary carrier, in which there is a tension at its center with a known load $B_x$. This load is the horizontal tension of the wire, and then it is a constant equivalent to the horizontal component of the reaction in the union. To calculate the vertical component, it is supposed initially, that the weight of droppers is zero, ($P^b_{pi} = 0$), because the length of the droppers is unknown. Under the assumption that each dropper is supporting its own weight, half of the weight of the contact wire located between two consecutive droppers, and some constant weight $P^b_g$:

$$R^b_i = n_c \cdot q \cdot \frac{(x^b_{i+1} - x^b_{i-1})}{2} + P^b_g + P^b_{pi} \ , \qquad i = 1...n_b \tag{1}$$

And vertical reactions in the unions are:

$$B_{1y} = \sum_{i=1}^{n_b} R_i^b + p_b \cdot L_b + P_e + \frac{P_c}{2} \tag{2}$$

$$B_{2y} = \sum_{i=1}^{n_b} R_i^b + p_b \cdot L_b + P_e + \frac{P_c}{2} \tag{3}$$

### 3.2 Calculation of the loads at the droppers of the main carrier

As the previous case, to determine the reactions at the supports, It is necessary to know the value of the loads at the droppers. Thus, it is supposed that each dropper is supporting its own weight and half of the weight of the contact wire between two consecutive droppers. Furthermore, it is supposed also that, initially, the weight of the droppers is zero ($P_{pi}^a = 0$), because at that point their length are unkonwn.

If the contact wire has a parabolic shape with a central deviation $f_c$, which produces a counterload $q'$. Thus, the following expression is obtained:

$$f_c = \frac{q'}{2 \cdot T_c} \cdot \frac{(x_{na}^a - x_1^a)^2}{4} \;,\; q' = \frac{8 \cdot T_c \cdot f_c}{(x_{na}^a - x_1^a)^2} \tag{4}$$

This counter load is compensated an overload in the extreme droppers, and thus the total load of the droppers of the main carrier is obtained.

### 3.3 Calculation of the reactions in the supports of the main carrier

If the balance for main carrier is considered, this one will be subject to its own weight, loads of the droppers which are suppose known, and reactions at the supports $A_1,\ A_2$ and reactions at the unions $B_1,\ B_2$ Thus, the components of these reactions will be, for $A_1$: $A_x,\ A_{1y}$, for $A_2$: $A_x,\ A_{2y}$, for $B_1$: $B_x,\ B_{1y}$, and for $B_2$: $B_x,\ B_{2y}$.

Of these values, at the beginning it is only known the tension at the secondary carrier $B_x$, and the complete reaction at $A_1$, that is, $T_1$ which corresponds to the tension at the compensating pulley. Then, to calculate the component $A_{1y}$ considering the balance equation of moments with respect to support $A_2$, the following expression is used

$$C \;=\; f_{B2} - f_{B1} \tag{5}$$

According with Figure 2, the resulting moment of weight, droppers and reactions of main carrier with respect to support $A_2$ is

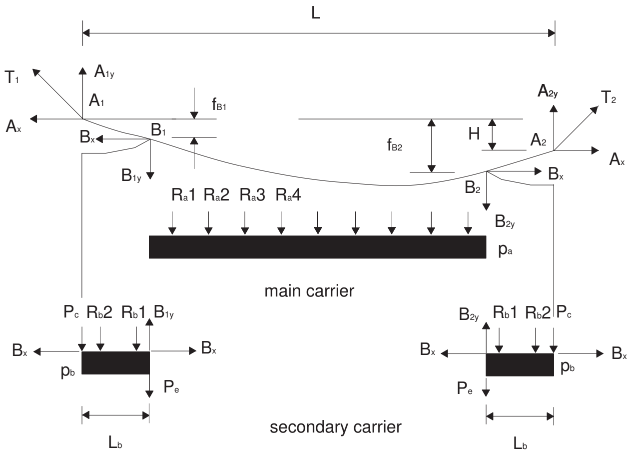



Figure 2: Diagram of free solid for the main and secondary carriers

$$M_{pA_2} = \sum_{i=1}^{i=n_a} R_i^a \cdot (L - x_i^a) + \frac{p_a \cdot L^2}{2} + B_{y1} \cdot (L - L_b) + B_{y2} \cdot L_b + B_x \cdot C \quad (6)$$

Initially, it is unknown the vertical position of the unions $f_{B1}, f_{B2}$, and then $C$, because this position depends of the deformation of the main carrier, which is unknown, due to that this problem is hyperstatic, then to obtain a first approach to the reactions, $C$ can be considered as zero, and then

$$H = H_{c1} - H_{c2} \quad (7)$$

Considering the equation of forces momentum equilibrium applying on the main carrier with respect of support $A_2$, and taking into account that tension at the main carrier in left support is known:

$$\sum M_{A2} = 0 \,, \quad A_{1y} \cdot L - A_x \cdot H - M_{pA2} = 0 \quad (8)$$

$$T_1 = \sqrt{A_x^2 + A_{1y}^2} \,, \quad A_x = \sqrt{T_1^2 - A_{1y}^2} \quad (9)$$

To calculate $A_{1y}$, the equation (9) is replaced into (8), and then, removing radicals and sorting terms obtaining a second order equation in $A_{1y}$

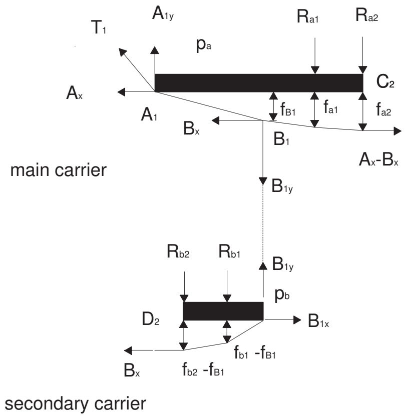



Figure 3: Diagram of free solid for two portions of the carriers

$$A_{1y}^2 \cdot (L^2 + H^2) - A_{1y} \cdot (2 \cdot M_{pA2} \cdot L) - T_1^2 \cdot H^2 + M_{pA2}^2 \;=\; 0 \tag{10}$$

From where the values of $A_{1y}$ and $A_x$ are obtained.

### 3.4 Calculation of deviations and lengths of the droppers of the main carrier

It is needed to know, at the first place, the deviation in the union with respect to the general system reference. According to Figure 3, and considering the part of the carrier between support $A_1$ and the union $B_1$, and taking the momentum with respect the point $B_1$, the following expression is obtained:

$$\sum M_{B1} = 0 \ , \qquad f_{B1} = \frac{1}{A_x} \cdot (A_{1y} \cdot L_b - \frac{p_a \cdot L_b^2}{2}) \tag{11}$$

Next, the deviation at the right union $B_2$ can be obtained:

$$f_{B2} = \frac{1}{A_x} \cdot (A_{2y} \cdot L_b - \frac{p_a \cdot L_b^2}{2}) + H \tag{12}$$

Again, according to Figure 3, and taking the momentum with respect to the position $C_i$ of each dropper:

$$\sum M_{Ci}=0\ ,\ f_i^a=\frac{1}{A_x+B_x}\cdot[A_{1y}\cdot x_i^a-\frac{p_a\cdot(x_i^a)^2}{2}-\sum_{j=1}^{j=i}R_j^a\cdot(x_i^a-x_j^a)\ -$$

$$B_{1y}\cdot(x_i^a-L_a)+B_x\cdot f_{B1}]\ ,\ i=1\dots n_a \tag{13}$$

And length and weight of these droppers will be:

$$L_{pi}^a=H_{c1}+f_{ci}^a-f_i^a\ ,\qquad P_{pi}^a=L_{pi}^a\cdot S\cdot\gamma\ ,\qquad i=1...n_a \tag{14}$$

### 3.5 Calculation of the deviations and lengths of the droppers of the secondary carrier

The corresponding deviation for a dropper situated in the secondary carrier is calculated in a similar way of that of the main carrier, taking the momentum with respect the generic point $D_i$, which represents the position for each dropper, and adding the deviation in the union, we obtain the following expression for the left secondary carrier:

$$\sum M_{Di}=0\ ,f_i^b=\frac{1}{B_x}[(B_{1y}-P_e)\cdot x_i^a-\frac{p_b\cdot(x_i^a)^2}{2}-\sum_{j=1}^{j=i}R_j^b\cdot(x_i^b-x_j^b)]+f_{B1}\ ,$$

$$i=1\dots n_b \tag{15}$$

In a similar way for the right secondary carrier:

$$f_i^b=\frac{1}{B_x}[(B_{2y}-P_e)\cdot x_i^a-\frac{p_b\cdot(x_i^a)^2}{2}-\sum_{j=1}^{j=i}R_j^b\cdot(x_i^b-x_j^b)]+f_{B2}\ ,i=1\dots n_b$$

Length and weight of droppers situated in the left secondary carrier will be:

$$L_{pi}^b=H_{C1}-f_i^b\ ,\qquad P_{pi}^b=L_{pi}^b\cdot S\cdot\gamma\ ,\qquad i=1\dots n_b \tag{16}$$

And length and weight of droppers situated in the right secondary carrier will be:

$$L^b_{pi} = H_{C2} - f^b_i \ , \quad P^b_{pi} = L^b_{pi} \cdot S \cdot \gamma \ , \quad i = 1 \ldots n_b \tag{17}$$

Now, the new value of vertical distance between $B_1$ and $B_2$ is defined:

$$C = f_{B2} - f_{B1} \tag{18}$$

For the new lengths, weight of droppers and distance $C$, it is necessary to calculate again the new values of vertical component of reactions at the unions, refered as $B'_{1y}$ and $B'_{2y}$, and the supports, called $A'_{1y}$ and $A'_{2y}$, according with the previous considerations.

Finally, the convergence of the algorithm, according with a parameter $\epsilon$, meaning the relative error allowed for the calculation, is checked. Then, the following comparison is carried out:

$$\left| \frac{A_{1y} - A'_{1y}}{A'_{1y}} \right| \ \leq \ \epsilon \quad \wedge \quad \left| \frac{B_{1y} - B'_{1y}}{B'_{1y}} \right| \ \leq \ \epsilon \tag{19}$$

In an affirmative case, the values of $A_{1y}$ and $B_{1y}$ are correct, and also the values for the deviations and lengths of droppers so calculated. In case of this conditions does not verify, these value will not be correct, and then it is necessary another iteration, where $A_{1y} = A'_{1y}$, and $B_{1y} = B'_{1y}$, repeating again the process until it is satisfied the condition expressed in equation 19.

Once finalized the mechanical calculation of the span, it is possible renumbering again the droppers with the corresponding lengths and deviations in the general system.

## 4 The software tool

The algorithm presented in the previous section has been implemented as part of a software tool, called CALPE. It has been written on an object-oriented database system with a visual interface under Windows 98. This framework is supported in the Visual FoxPro (© Microsoft) environment, and it is currently used by RENFE, the Spanish company of railways, in the development of its electrical catenary systems.

The tool, whose current user interfaces are in Spanish, consists of a menu, where it is possible to choose several options, among them the maintenance of the database system, designed with several files implementing the different tables of a relational database system following a previously designed entity-relation scheme. These tables implement the different components of the two catenary models described in this paper, the normal and the

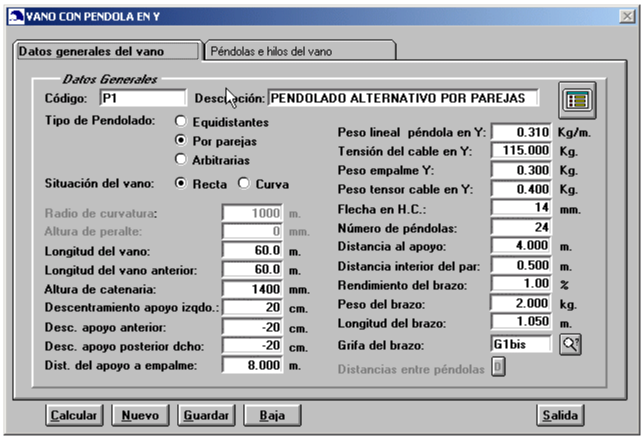



Figure 4: Data Input

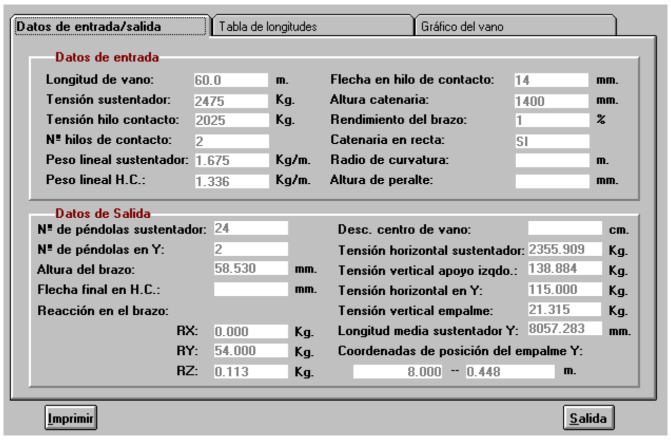
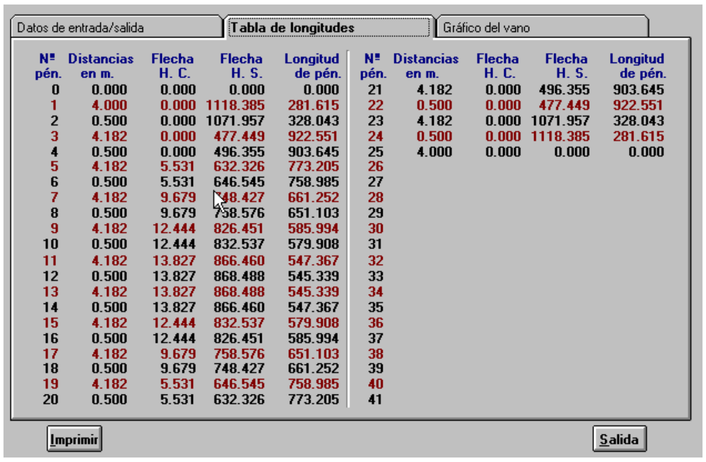



Figure 5: Input/output window

stitched span, and these models of catenary wires are represented by tables implementing different types of wires (carrier and contact), droppers, etc.

The main procedure in the tool is the design of the catenary. There are two procedures implementing this design, the so called "*normal*", in which it is implemented the algorithm presented in [2], and the "*stitched*", implementing the algorithm here presented. This procedure is also divided into two parts, defined each one of them over a window in a friendly user inter-




| Nº pén. | Distancias en m. | Flecha H. C. | Flecha H. S. | Longitud de pén. | Nº pén. | Distancias en m. | Flecha H. C. | Flecha H. S. | Longitud de pén. |
|---|---|---|---|---|---|---|---|---|---|
| 0 | 0.000 | 0.000 | 0.000 | 0.000 | 21 | 4.182 | 0.000 | 496.355 | 903.645 |
| 1 | 4.000 | 0.000 | 1118.385 | 281.615 | 22 | 0.500 | 0.000 | 477.449 | 922.551 |
| 2 | 0.500 | 0.000 | 1071.957 | 328.043 | 23 | 4.182 | 0.000 | 1071.957 | 328.043 |
| 3 | 4.182 | 0.000 | 477.449 | 922.551 | 24 | 0.500 | 0.000 | 1118.385 | 281.615 |
| 4 | 0.500 | 0.000 | 496.355 | 903.645 | 25 | 4.000 | 0.000 | 0.000 | 0.000 |
| 5 | 4.182 | 5.531 | 632.326 | 773.205 | 26 | | | | |
| 6 | 0.500 | 5.531 | 646.545 | 758.985 | 27 | | | | |
| 7 | 4.182 | 9.679 | 748.427 | 661.252 | 28 | | | | |
| 8 | 0.500 | 9.679 | 758.576 | 651.103 | 29 | | | | |
| 9 | 4.182 | 12.444 | 826.451 | 585.994 | 30 | | | | |
| 10 | 0.500 | 12.444 | 832.537 | 579.908 | 31 | | | | |
| 11 | 4.182 | 13.827 | 866.460 | 547.367 | 32 | | | | |
| 12 | 0.500 | 13.827 | 868.488 | 545.339 | 33 | | | | |
| 13 | 4.182 | 13.827 | 868.488 | 545.339 | 34 | | | | |
| 14 | 0.500 | 13.827 | 866.460 | 547.367 | 35 | | | | |
| 15 | 4.182 | 12.444 | 832.537 | 579.908 | 36 | | | | |
| 16 | 0.500 | 12.444 | 826.451 | 585.994 | 37 | | | | |
| 17 | 4.182 | 9.679 | 758.576 | 651.103 | 38 | | | | |
| 18 | 0.500 | 9.679 | 748.427 | 661.252 | 39 | | | | |
| 19 | 4.182 | 5.531 | 646.545 | 758.985 | 40 | | | | |
| 20 | 0.500 | 5.531 | 632.326 | 773.205 | 41 | | | | |

Figure 6: Table of droppers

face. The first one consists of the input of the different data types, selected among the previously introduced components in the database system, and some other new data types. In fig. 4 an example of data introduction over a window of the tool is presented.

After that, the output results are presented in another window, again divided into three sections, the first one showing the main results of the calculations. The window with the collection of input/output data is shown in Figure 5.

Finally the table of lengths of the droppers is shown in two windows, the first one (see Figure 6) where the data are presented in an array of data, and the second one (see Figure 7), where the data are presented in a picture showing the shape of the catenary.

## 5 Conclusions and Future Work

In this work an algorithm to calculate an electric power line (catenary) for railways has been presented, considering an advanced model of the wire called the stitched catenary, in which the carrier is divided into a main carrier and two secondary carriers. This model have the advantage of a lower variation of the stiffness of the span.

In a normal model of the catenary, it is known that the deformation of the catenary due to the up pressure of the pantograph over the contact wire is a

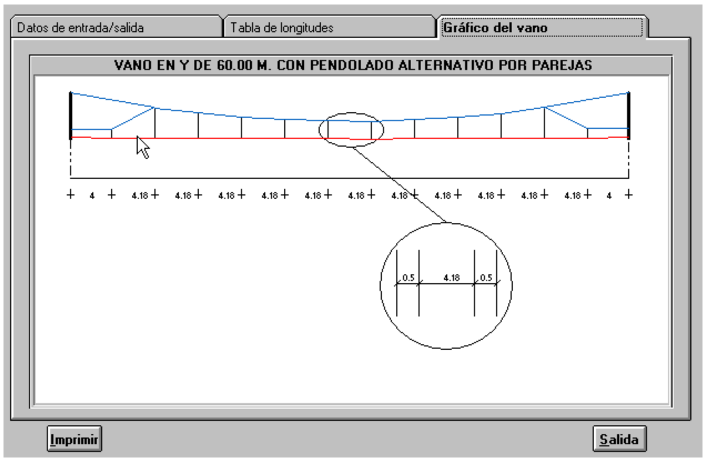



Figure 7: Picture of droppers

little lower in the center of the span than in the supports, and this may cause oscillations in the pantograph. Thus, this problem is avoided in the model here presented, by allowing to the contact wire to have a little deviation at the center of the span, and placing a false carrier in the supports, obtaining in this way a more uniform stiffness along the span.

This algorithm, and the algorithm with the calculations relative to the normal span case, has been implemented in a software tool (CALPE), developed by the University of Castilla-La Mancha, and currently used by RENFE (spanish company of railways) to design the catenaries of its railways, obtaining better quality lines.

Nowdays, it is under study more considerations related with the stiffness of the span, covering the static case, in which the force of the pantograph over the contact wire is punctual [1].

The future work will take into account the dynamic case, in which we will study the force of the pantograph taking into account that this pantograph is in motion along the wire at a high speed. The knowledge of the stiffness will allow us a better structure of the electrical system of the catenary, due to a bigger correctness in the deviation of the span, and a bigger correctness, also, in the calculation of the tension and length of the carrier, having an uniform stiffness along the span.

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